Advanced Industrial Organization
Final Exam
3-hour, closed book

Written Exam at the Department of Economics, Winter 2019

December 19, 2019

Exercise 1: The Diamond Paradox

N firms produce a perfectly homogenous good at zero marginal cost, $c = 0$. The firms compete in prices, $p_1, \ldots, p_N$ (Betrand), in a one-shot game. There is a unit mass of consumers, each demanding at most one unit of the good. Each consumer knows the price charged by one random store. Consumers can learn the price of other stores by randomly searching one store at a time and learn its price. For each store a consumer searches, a search cost, $s > 0$, is incurred. Consumers have a reserve price $r > 0$ above which demand is zero. That is, consumers will consume the cheapest product they have searched (if multiple firms tie, consumers randomize between them) with a price $p \leq r$. If a consumer has searched only products with $p > r$, they will leave the market without purchasing. Consumers hold an expectation about market prices, $p_1^E, \ldots p_N^E$ which they use to optimally decide how many products to search. In equilibrium we require that 1) the actual prices $p_1, \ldots, p_N$ form a Bertrand Nash equilibrium between firms, 2) consumer expectations are correct so $p_i^E = p_i \forall i \in \{1 \ldots N\}$ and 3) consumers search and consume optimally given their expectations.

Question 1a

Show that with $s > 0$, there is an equilibrium in which all firms firms charge $p_1 = p_2 = \ldots p_N = r$ (the monopoly price). (Hint: Start by assuming $p_1 = \cdots = p_N = r$ is an equilibrium and consider the incentives for each firm to deviate)

Answer: Assuming that all firms charge $p_1 = p_2 = \ldots p_N = r$, and consumers expecting this, it is clear that consumers have no incentive to search. Therefore, a firm undercutting to, say $r - \epsilon$, will not gain any new market share, but will on the other hand earn less revenue on the share of consumers consuming his product. Increasing the price above r will reduce profits to zero, because no consumer will buy. Thus, firms have no reason to deviate from r.Therefore, $p_1 = p_2 = \ldots p_N = r$ is an equilibrium.

Question 1b

Show that the competitive outcome $p_1 = p_2 = \ldots p_N = c$, is not an equilibrium with $s > 0$. (Hint: Start by assuming $p_1 = \cdots = p_N = c$ is an equilibrium and consider the incentives for each firm to deviate)

Answer: Starting from $p_1 = p_2 = \ldots p_N = c$ no consumers will search. Consider now firm i raising its price to $c + s/2$. The net gain from search for consumers starting at firm i is $(p_i - c - s) = ((c + s/2) - c) - s = s/2 - s = -s/2 < 0$. Therefore, consumers still do best by not searching and consuming product i. From the firm's perspective, this means that by raising its price, it is increasing its profit, and hence, $p_1 = p_2 = \cdots = c$ is not a Nash equilibrium.

Question 1c

Show that the equilibrium in Question 1a is the unique pure strategy equilibrium. (Note: there is no mixed strategy equilibrium in the game. This you don't have to show). (Hint: Start from an arbitrary, potentially asymmetric, pure strategy equilibrium $p_1, \ldots p_N$ and consider the incentives of the lowest price firm)

Answer: Starting from an arbitrary equilibrium $p_1, p_2, \ldots p_N$. Consider a firm charging the lowest price, $\underline{p} = \min(p_1, p_2, \ldots p_N)$ and consider the outcome of changing its price to $\overline{p} = \min(\underline{p} + s/2, r)$. Consider now a consumer starting at firm i. The largest possible gain from search is $(\overline{p} - \underline{p}) - s \leq \underline{p} + s/2 - \underline{p} - \underline{s} = -s/2 < 0$. Therefore, a consumer starting at product i will not search and therefore firm i can increase its profit from increasing its price. We have therefore reached a contradiction. Unless the low price firm charges $p_i = r$,it has an incentive to increase its price.

Exercise 2: A Model of Sales

N firms produce a perfectly homogenous good. The firms compete in prices, $p_1, \ldots, p_N$ (Betrand), in a one-shot game. Firms produce q units at total cost $C(q) = F$ with $F > 0$ being a fixed cost, and with zero marginal costs $C'(q) = 0$. The demand consists of a mass $I \geq 0$ of informed consumers and a mass $M \geq 0$ of uninformed consumers. We let $U = M/N$ denote the mass of uninformed consumers per firm. Each consumer demands at most one unit of the good. There is a reserve price $r > 0$ above which no consumer will purchase. Informed consumers will buy the cheapest product, as long as its price is below r, and uninformed consumers buy a random product, as long the price of this product is below r. If multiple firms tie at the lowest price, each informed consumer will randomize between them such that the tying firms split the informed consumers I equally.

Question 2a

Let p_i be the price of firm i, and p_{-i} a vector of prices of all its competitors. Write up a profit function for firm i, $\pi_i(p_i, p_{-i})$, as a function of p_i and p_{-i}.

Answer: Let p_{win} be the lowest price, and $N_w = \sum_{i+1}^{N} 1_{\{p_i = p_{win}\}}$ be the number of firms tying at the lowest price. We can write the profit as

$$\pi_i(p_i, p_{-i}) = \begin{cases} p_i(I/N_{win} + U) - F & \text{if } p_i \leq \min(p_{-i}) \\ p_i U - F & \text{if } p_i \in (\min(p_{-i}), r] \\ 0 & \text{else} \end{cases}$$

Question 2b

Holding the number of firms, N, fixed. Derive an expression for p^*, which is the lowest price any firm is willing to charge. How does p^* depend on N, I and U?

Answer: Firms can always submit $p_i = r$, lose, and earn $rU - F$. The lowest price firms will charge, p^*, is such that the firm will be indifferent between winning at price p^* or losing at price r. That is

$$p^*(I + U) = rU$$
$$\Leftrightarrow p^* = \frac{rU}{I + U}$$

It is clear that p^* is increasing in U and r and decreasing in I. This is intuitive since higher U and r increases the profit from selling only to informed consumers, while a higher I makes undercutting more attractive. The effect of N is seen by realizing that $U = \frac{M}{N}$ so for fixed M a higher N will decrease U and therefore, the lowest price firms are willing to charge falls.

Question 2c

Construct an argument for why the game does not possess a *symmetric* Nash equilibrium in pure strategies, where all firms charge the same price.

Answer: No firm will charge a price below p^* or above r, so any symmetric pure strategy equilibrium will have to lie in this interval. Suppose it is an equilibrium that all firms charge $p = p_1 = p_2 = \cdots = p_N$. But: if $p > p^*$ then any one firm can increase profit by charging $p - \varepsilon > p^*$ for some small $\varepsilon > 0$ and capture the whole market. If $p = p^*$ on the other hand, then firms can increase profit by deviating to r. This is because by the definition of p^*, firms are indifferent between winning alone at p^* or losing at r. But this implies that when multiple firms tie at p^* it is more profitable for firms to charge r, because tying on p^* will earn $(I/N_{win} + U)p^* < Ur$. Lastly, as a side-note (this comment is not expected or required by students), there exist N asymmetric pure strategy equilibrium where one firm charges p^* and the remaining firms charge r (and where the only difference between the N equilibria is in the identify of the winning firm).

Question 2d

Assuming that the opponents of firm i play each price with a mixing density $f(p)$ over the support $[p^*, r]$, write up the profit maximization problem of firm i, when firm i has to select an optimal mixing distribution. Solve for the equilibrium mixing cdf, $F(p)$.

Answer: The constrained maximization problem writes

$$\max_{f(p)} \int_{p^*}^{r} \left\{ (1 - F(p))^{N-1} p\,(I + U) + \left[1 - (1 - F(p))^{N-1}\right] pU \right\} f(p)dp$$

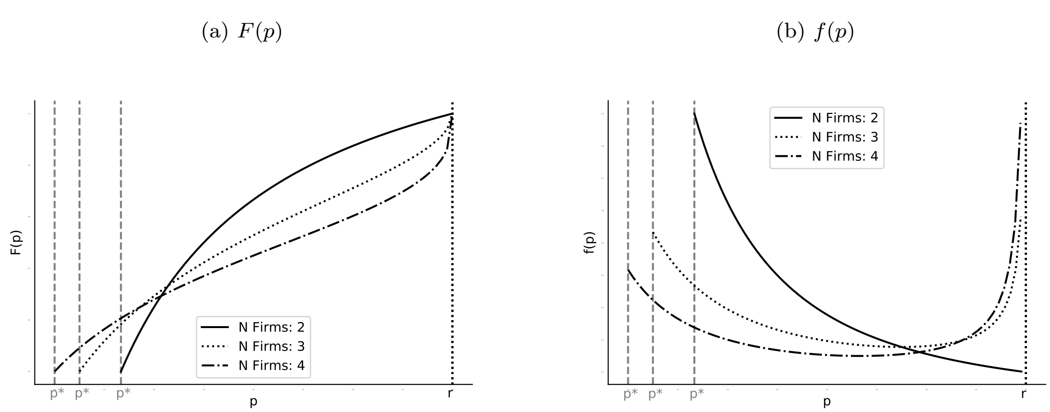
s.t. $f(p) \geq 0$ $\int_{p^*}^{r} f(p)dp = 1$. Where the firm face the constraint that the mixing distribution must in fact be a distribution, that is non-negative density and integrate to 1. To solve for $F(\cdot)$ we use the principle of indifference, that is, $F(\cdot)$ is such that all prices $p \in [p^*, r]$ yield the same expected profit. Because any one firm can always deviate to r and serve the uninformed consumers, this common profit equals Ur. Therefore, it must be the case that

$$\begin{aligned}
(1 - F(p))^{N-1} p\,(I + U) + \left[1 - (1 - F(p))^{N-1}\right] pU &= Ur \\
(1 - F(p))^{N-1} \left[p\,(I + U) - pU\right] \Leftrightarrow &= Ur - Up \\
\Leftrightarrow (1 - F(p))^{N-1} &= \frac{U(r-p)}{pI} \\
\Leftrightarrow 1 - F(p) &= \left(\frac{U(r-p)}{pI}\right)^{\frac{1}{N-1}} \\
\Leftrightarrow F(p) &= 1 - \left(\frac{U(r-p)}{pI}\right)^{\frac{1}{N-1}}
\end{aligned}$$

Question 2e

Figure 1 (Below) plots the equilibrium pdf, $f(p)$, and the corresponding cdf $F(p)$ for $N = 2, 3, 4$. The lower boundary of the support, p^* is indicated by vertical dashed lines, and the upper boundary of the support, r, is indicated by a vertical dotted line. How does increased competition affect the propensity of firms to submit prices in the bottom, middle and upper range? Provide an intuitive explanation for the relation between competition and the shape of the probability mixing distribution.

Figure 1: Equilibrium Price Distribution and Number of Firms



Answer: Increasing the number of firms have countervailing effects on prices. On the one hand, the mass of uninformed consumers per firm $U = M/N$ decreases, yielding in turn a decrease in the average profit rU.

Therefore, the lowest price firms are willing to charge, p^*, decreases in N. So more firms leads to a decrease in the lower boundary of the support and thus more mass at low prices (business stealing motive) and a lower average profit overall. On the other hand, given $F(\cdot)$, the probability of winning at price p $(1 - F(p))^{N-1}$ decreases exponentially in N, which makes it more attractive extract profit from the uninformed consumers and put more mass in the top of the distribution. This is called the surplus appropriation motive. As a consequence, the mass in the top of the distribution also increases. Because more mass is put in the bottom and top of the distribution, the mass in the middle range will decrease. The business stealing and surplus appropriation effects are however not symmetric. While the probability of winning decreases exponentially in N, the profit from charging a high price is proportionally decreasing at rate $\frac{1}{N}$. Therefore, the surplus appropriation motive will dominate, and firms will be increasingly likely to submit a high price.

Exercise 3

This exercise will ask you questions about the paper "Measuring the Incentive to Collude: The Vitamin Cartels, 1990-1999" by Mitsuru Igami and Takuo Sugaya (WP, 2019). Maintaining the notation from the paper, we consider a set $\mathcal{I}$ of potentially colluding firms participating in repeated Cournot competition over an infinite time horizon, $t = 1, \dots \infty$. That is, in each period, and for all $i \in \mathcal{I}$, firm i chooses quantity $q_{i,t}$. Firms have differing and time-varying marginal costs, $c_{i,t}$. Firms have a common discount factor $\beta \in (0, 1)$. The firms in $\mathcal{I}$ face a linear inverse demand curve $P_t(Q_{car}, Q_{fri}) = \tilde{X}_t - \alpha (Q_{\mathcal{I}} + Q_{fri})$ where $\tilde{X}_t$ is a time varying demand shifter, $Q_{\mathcal{I}} = \sum_{i \in I} q_{i,t}$ is the total output of firms in $\mathcal{I}$ and Q_{fri} is the exogenous output from a set of non-colluding fringe firms. We assume that a collusive agreement amounts to all firms in $\mathcal{I}$, taking Q_{fri} as given, producing a quota $\overline{q}_{i,t}$ such that $\sum_{i \in \mathcal{I}} \overline{q}_{i,t} = \arg\max_Q \left(P_t(Q, Q_{fri}) - c_*\right) Q$ where c_* is the marginal cost of the industry leader (that is, the quotas maximize the aggregate profit of firms in $\mathcal{I}$, assuming they have marginal cost c_*). We assume that firms observe the quantities of each other with a $L = 3$ period lag. We say that non-compliance is confirmed in period τ if some firm did not produce $\overline{q}_{i,t}$ in period $\tau - L$ for the first time. We then suppose that firms agree to play the following equilibrium based on trigger strategies: If no non-compliance is confirmed before period τ then each firm sells $q_{i,\tau} = \overline{q}_{i,\tau}$. If some non-compliance is confirmed in some previous period, $s \leq \tau$ then each firm sells a static Nash equilibrium quantity $q_{i,\tau} = q^N_{i,\tau}$. Lastly, we define the optimal deviation quantity $q^D_{i,\tau}$ which is the best response to all firms in $\mathcal{I} \backslash i$ playing their collusive quotas $\overline{q}_{j \in \mathcal{I} \backslash i,\tau}$. $\overline{q}_{i,t}, q^N_{i,t}$ and $q^D_{i,t}$ gives rise to corresponding stage game profits $\pi^C_{i,t}, \pi^N_{i,t}$ and $\pi^D_{i,t}$ such that $\pi^D_{i,t} > \pi^C_{i,t} > \pi^N_{i,t}$. The paper structurally estimates the Incentive Compatibility Condition (ICC) for collusion to be sustainable in a market, as written in Equation (1)

$$\min_{i \in \mathcal{I}, \tau \geq t} \left(V^C_{i,\tau|t} - V^D_{i,\tau|t} \right) \geq 0 \tag{1}$$

Where $V^C_{i,\tau|t}$ and $V^D_{i,\tau|t}$ measures the expected net present value to firm i of respectively colluding and deviating in period $\tau \geq t$ with the expectation taken in period t and where the minimum is over the set of firms participating in the cartel, $\mathcal{I}$, and over all future periods $\tau \geq t$. Furthermore, $V^C_{i,\tau|t}$ and $V^D_{i,\tau|t}$ are defined as follows.

$$V^C_{i,\tau|t} = \sum_{s \geq \tau} \beta^{s-\tau} \pi^C_{i,s|t} \tag{2}$$

$$V^D_{i,\tau|t} = \sum_{s=\tau}^{\tau+2} \beta^{s-\tau} \pi^D_{i,s|t} + \sum_{s \geq \tau+3} \beta^{s-\tau} \pi^N_{i,s|t} \tag{3}$$

Question 3a

Provide an explanation for why the condition in Equation (1) is a necessary condition for collusion to be sustainable in a market.

Answer: The condition states that the net present value of playing a collusive price must exceed the value of deviating for all firms in the cartel, $i \in \mathcal{I}$ and in all future periods. The reason is that if the condition is violated for some firm, then all other firms will realize that this firm will want to deviate. This in turn will lead all other firms to deviate immediately. Similarly, if the condition is violated for some firm i at some future period τ then all firms will deviate in this period. Knowing this, all firms are in effect playing a finite horizon game, with period τ as the final period. In period $\tau - 1$, firms know that whether they collude or not, they will get the Nash outcome in period τ. They will therefore choose to deviate in period $\tau - 1$ which now becomes the new "final" period. In this way, the cartel unravels all the way back to period t.

Question 3b

The authors assume that firms can only observe the past actions of each other with a 3-period lag. Based on Equation (1), (2) and (3), please explain how an information lag affects the incentive to collude.

Answer: Looking at the equation it is clear that the collusion payoff $V^C_{i,\tau|t}$ is unaffected by the information lag. The deviation payoff $V^D_{i,\tau|t}$ is however increasing in the information lag, because a deviating firm will receive the unilateral deviation payoff $\pi^D_{i,s,|t}$ for more periods before being punished when the competing firms can detect deviation. An information lag therefore increases the incentive to deviate which in turn makes it harder to sustain collusion (the discount value β will need to be higher for Equation (1) to be satisfied)

Question 3c

Figure 7 (in appendix) plots empirical estimates of the collective (market) and firm-specific ICC for the Vitamin C market. Figure 8 plots the corresponding ICC for the market leader Roche in the Vitamin A, E, and Beta Carotene market. Based on Figure 7 and 8, please explain

- When does the estimated ICCs predict that a industry-level cartel is sustainable and not sustainable respectively? Why?

Answer: Figure 7 shows that prior to 1995 the ICC is positive and statistically significantly different from zero. Therefore, a cartel in the Vitamin C market was sustainable in this period. Between 1995-1997, the ICC is no longer significantly different from zero and therefore one cannot reject the hypothesis that a cartel is unsustainable in this period. We interpret therefore the model to predict a cartel breakdown between 1995 and 1997, which coincides with the actual breakdown. (A note which the students are not expected to make: For binary outcomes, we usually say a model predict 1 (collusion, say) if the probability of 1 exceeds 50%. But a confidence interval does not give us a probability distribution of the ICC - it doesn't tell us with what probability the ICC would be violated in a given year. What we are doing is therefore instead that collapse is predicted whenever it cannot be rejected, regardless of how likely it is)

- How do you think the model prediction of a potential breakdown fits reality in the four markets? (The Vitamin C cartel collapsed in 1995 while the others remained in effect until 1999)

Answer: The student here is simply expected to point out that the Vitamin C cartel is the only market in which the ICC becomes insignificantly different from zero. For all other markets, the model predict that a cartel is sustainable. Since the Vitamin C market is the only one of the four markets in which the cartel broke down before the litigation, this suggest that the model aligns well with reality.

Figure 9 (in appendix) plots simulations of the ICC for the Vitamin C cartel under various counterfactual scenarios

- Based on Figure 9, give your analysis of the likely cause of the Vitamin C cartel breakdown

Answer: The plot shows the development of the ICC under counterfactual scenarios in which either the demand didn't slow down, the supply from fringe (non-cartel firms) didn't grow or both. Scenario 1 is the dream scenario in which both the demand didn't slow down and the fringe supply didn't grow. Under this scenario, the cartel would have survived during the whole sample period. Scenario 2 shows that the growth in fringe supply would have tightened the ICC but not enough that it would be violated. If we instead consider scenario 3 where the demand slowed down but without fringe growth we see that the ICC tightens much more than under scenario 2, suggesting that demand growth is more important for the ICC than fringe supply, but not enough that it is violated. This shows that both fringe supply growth and demand slowdown contributed to the breakdown of the cartel, but none of the factors were strong enough to unilaterally cause the breakdown.

Question 3d

The paper considers the effect of a hypothetical merger in 1991 between BASF and Takeda (BASF did in fact acquire Takeda in 2001, after the cartel collapse). Because the firms had different marginal costs (Takeda had superior production capability relative to BASF) the authors must make an assumption about how the ex post marginal costs relate to the marginal costs of the merging firms. The authors consider the following relation

$$c^{post}_{basf,t} = (1 - \sigma) \times \min\left\{c^{pre}_{takeda}, c^{pre}_{basf}\right\} = (1 - \sigma) \times c^{pre}_{takeda}$$

Where $\sigma \in (0, 1)$ is a so-called "efficiency gain" parameter

- Provide an interpretation of the above relation. Do you find this plausible?

Answer: The above relation assumes that the merging firm is at least able to achieve the cost level of the lowest cost firm. This assumption is justified in the paper by the fact that after Takeda and BASF actually merged in 2001, BASF shut down its own facilities and switched all production to Takeda's plants. On top of this, there is potential of efficiency effect. This is reflected in the efficiency parameter, σ. The paper does not discuss whether it is plausible that the two firms could obtain efficiency gains. One way a merger could lead to an efficiency gain could be through the transfer of knowhow and productive technologies. This could for example be the case if the high cost firm has a patent on a cost reducing technology which could complement the technology of the low cost firm. Then the resulting marginal cost of the merged firm would be lower.

- Based on Figure 10 (in appendix), give your analysis of the effect of concentration on cartel stability. How do cost synergies impact the effect of a merger? What is the intuition for this?

Answer: Figure 10 shows that mergers increase the cartel stability all else equal. Under the counterfactual in which $\sigma = 0$, the cartel survives longer. However, if the merger is associated with significant efficiency gains, the merger reduces cartel stability. So the effect of cost synergies is to destabilize the cartel. The reason for this is that when the cost reduction is significant, the resulting set of competing firms will have increasingly asymmetric costs. Specifically, because the firm resulting from the merger will have much lower costs than its competitors, its incentive to participate in the cartel will decrease, both because the potential gain from participating in the cartel relative to competing in static Nash competition as the lowest cost firm will decrease, and because the one-off deviation payoff increases when costs decrease. Therefore, a merger can have counteracting effects on cartel stability.

Appendix

Figure 7: Collective and Individual ICCs (Vitamin C)

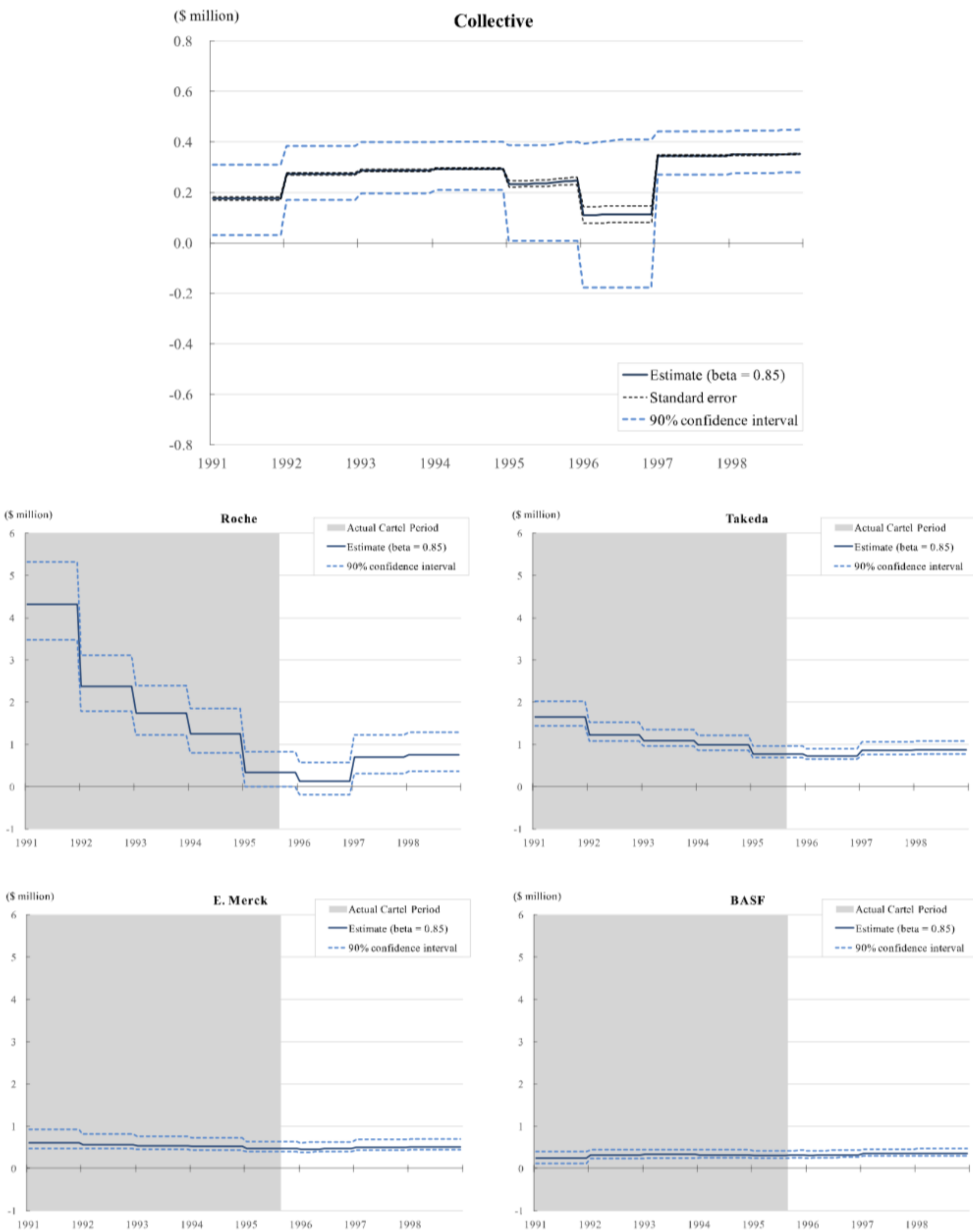


Figure 8: Roche's ICCs (Vitamins A and E, and Beta Carotene)

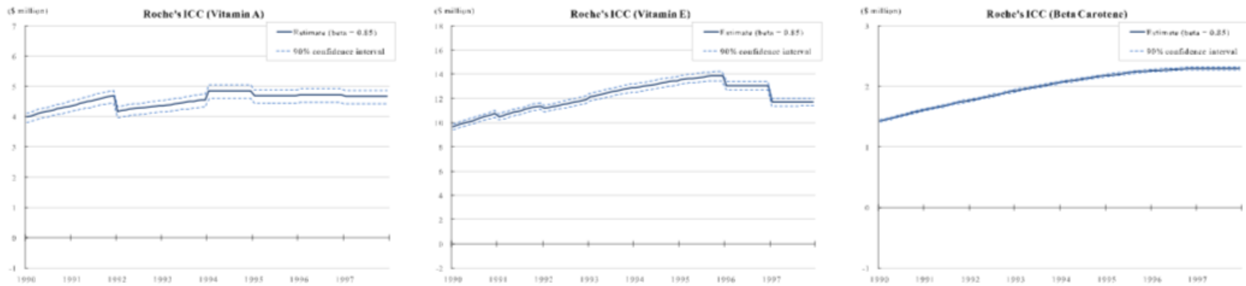


Figure 9: Effects of Demand and Supply on Roche's ICC (Vitamin C)

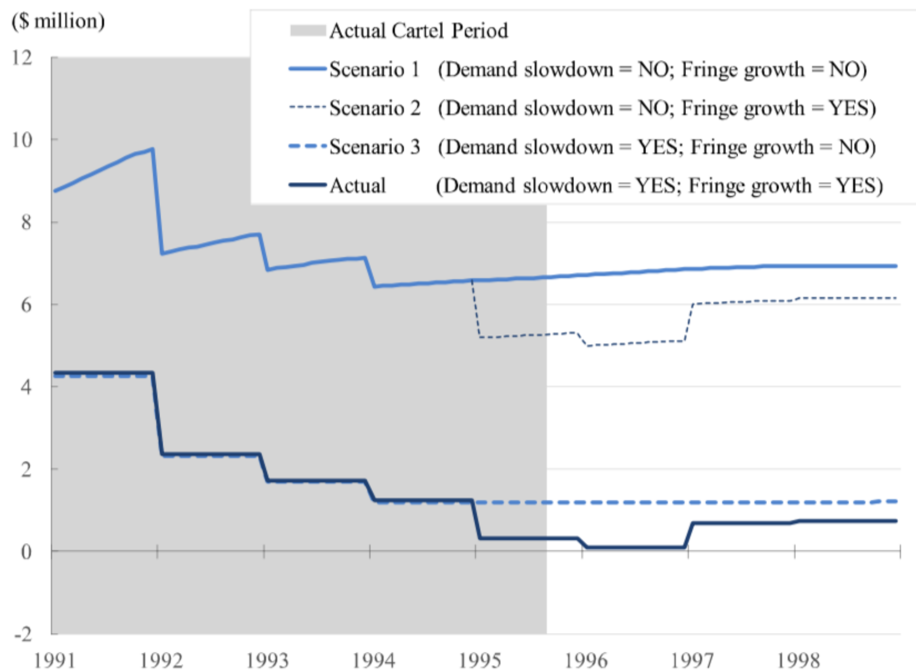
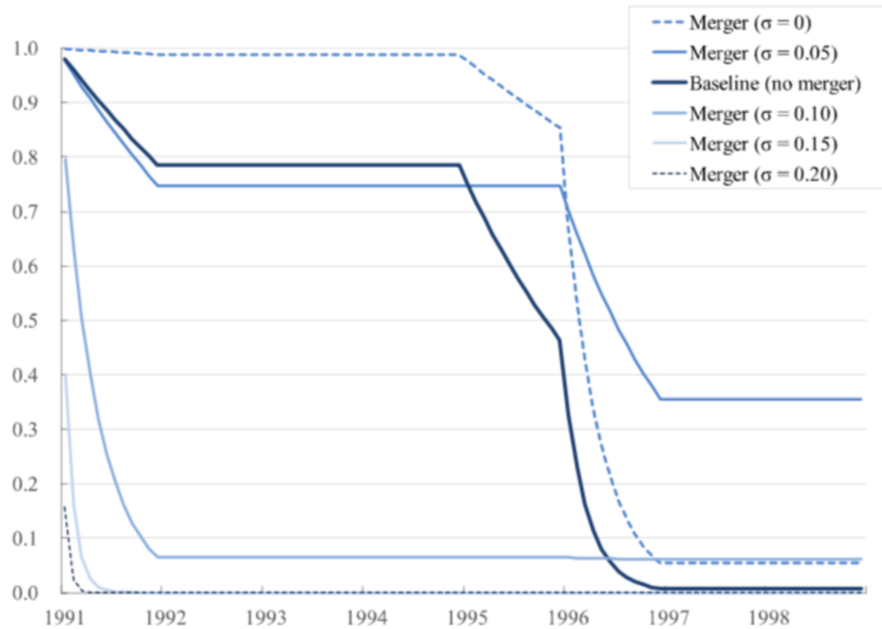


Figure 10: If BASF Had Acquired Takeda's Vitamin C Assets before 1991



Note: Each graph in Figure 10 shows on the y-axis the probability (computed from block bootstrap simulations) that the cartel would have survived beyond a given date, under the assumptions given in the legend.